

# MAJORANA – LIKE MODELS IN THE PHYSICS OF NEUTRAL PARTICLES\*

VALERI V. DVOEGLAZOV<sup>†</sup>

*Escuela de Física, Universidad Autónoma de Zacatecas  
Antonio Dovalí Jaime s/n, Zacatecas 98068, ZAC., México  
Internet address: VALERI@CANTERA.REDUAZ.MX*

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**Abstract.** Due to the standard electroweak model we have become accustomed to think about a neutrino  $\nu$  and its antineutrino  $\bar{\nu}$  as distinct particles. However, it has long been recognized that the apparent distinction between them may be only an illusion. Implying these words of Prof. B. Kayser we give an alternative insight in the physics of neutral particles (neutrino and photon). The proposed formalism which is based on the Majorana ideas could also be useful for deeper understanding of the nature of electron.

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The standard electroweak model tells us that neutrino can be only left-handed and antineutrino can be only right-handed. Neutrino and its antineutrino are different particles in the framework of conventional approaches. However, the standard model, which is built on the base of the Dirac construct for fermions and of the gauge principle, is not able to explain us, why the mirror image of the  $\beta$ -decay does not occur in Nature, where are ‘missing’ right-handed neutrino and left-handed antineutrino? In this talk I try to use a viewpoint based on the Majorana ideas in order to understand mathematical origins of these facts. First of all, let me discuss the present situation in the physics of neutral particles (as well as in Physics itself). At the moment we have:

- the solar neutrino puzzle;
- the negative mass squared problem (*cf.* with the old ITEP result (1987));
- the atmospheric neutrino anomaly;
- speculations on the possibility of the neutrinoless double  $\beta$ -decay;
- the experimental evidence for tensor coupling in the decay  $\pi^- \rightarrow e^- + \bar{\nu}_e + \gamma$  (as well as in the decays of  $K$  mesons);
- the dark matter problem;

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<sup>†</sup> On leave of absence from *Dept. Theor. & Nucl. Phys., Saratov State University, Astrakhan-skaya ul., 83, Saratov RUSSIA*. Internet address: dvoeglazov@main1.jinr.dubna.su

- the problem of  $\gamma$ -ray bursts;
- candidate events for neutrino oscillations (LANL, April 1995), what, according to present-of-day ideas, could lead to the conclusion of existence of the neutrino mass and, probably, of the fourth generation;
- in addition, the spin crisis in QCD.

Furthermore, there are several theoretical puzzles in basic structures of the quantum field theory (QFT):

- In classical physics antisymmetric tensor field is transversal; on the other hand it was proved that ‘after quantization’ the antisymmetric tensor field is longitudinal. Does this fact signify that we must abandon the Correspondence Principle?...
- The problem of the indefinite metric in QFT. Nobody appears to understand its physical sense. Moreover, from a viewpoint of mathematics it is rather obscure construct.
- Finally, the renormalization idea, which “would be sensible only if it was applied with finite renormalization factors, not infinite ones (one is not allowed to neglect and [to subtract] infinitely large quantities)”. These words are not of mine but the words of Prof. Dirac presented in his last lectures [1a,p.4-5].

While I am not going to answer all these questions in the present talk, but quite a number of these problems seems to me to be overwhelming if compare with only three black spots (black-body radiation, photoeffect and  $\alpha$ - particle scattering) on the cloudless sky of the classical physics in the end of the nineteenth century.

Various models have been proposed for explanation of the present situation. I list some of them:

- Non-zero electric charge of neutrino [2]. These models are based on the realization of the old Einstein’s idea of the electric charge dequantization.
- Existence of the mirror matter (as a particular case, of the mirror photon with electric charge and/or mass), ref. [3, 4].
- Neutrino theory of light (*e.g.*, ref. [5]). “...In view of the neutrino theory of light, photons are likely to interact weakly also, apart from the usual electromagnetic interactions... This assumed photon-neutrino weak interaction, if it exists, will have important bearing on astrophysics...”
- Tachyonic neutrinos, ref. [6].
- Introduction of the Evans-Vigier longitudinal  $B(3)$  field of electromagnetism, ref. [7]. Let me remind that many physicists (including Dirac [1b,p.32]) have risen the problem of longitudinal modes, *e.g.*, [8, 9]. In my opinion, recent preprints and papers [10]-[15] proved that the problem exists.
- The use of other representations of the extended Lorentz group (namely, the doubled representations [16]), see, *e.g.*, ref. [17].

Several of these models are certainly exotic if not say ‘crazy’. But, I hope, you remember the known saying of the great physicist of the twentieth century: “The idea is crazy. Is it *sufficiently* crazy to be correct?”

Our aim with this talk (and with the recent series of our papers) is to understand the Majorana ideas [18] of constructing the theory of neutral particles and to propose models based on these principles with possible application to neutrino physics. The work in this direction has been started in [19]-[21]. Apart from these papers I

would also like to mention several papers of Barut and Ziino which deal with similar matters [22, 23].

First of all, let me touch the Dirac case:

$$[i\gamma^\mu \partial_\mu - m\mathbf{1}] \Psi(x) = 0 \quad . \quad (1)$$

Everybody knows the physical sense of the spinorial basis (in the standard representation of  $\gamma$  matrices)

$$u^{(1)}(\overset{\circ}{p}^\mu) = \sqrt{m} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad u^{(2)}(\overset{\circ}{p}^\mu) = \sqrt{m} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad (2)$$

$$v^{(1)}(\overset{\circ}{p}^\mu) = \sqrt{m} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad v^{(2)}(\overset{\circ}{p}^\mu) = \sqrt{m} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \quad (3)$$

( $\overset{\circ}{p}^\mu$  should be referred to the frame with the momentum  $\mathbf{p} \rightarrow \mathbf{0}$ ). The equation (1) describes eigenstates of the Charge Operator. One can attach particle-antiparticle interpretation of bispinors (2,3). But, if apply the Majorana *ansatz* (in the coordinate representation; signs  $\pm$  are referred to left and right projections  $\Psi_\pm = \frac{1}{\sqrt{2}}(1 \pm \gamma_5)\Psi$ ):

$$\Psi_-(x) = \mathcal{C}_{[1/2]} \Psi_+^*(x) \quad (4)$$

one can obtain neutral particles. Why? This question has been analyzed in ref. [24]. A review of topics connected with this interpretation of the Majorana ideas could be found in ref. [25].

I am going to consider models based on the following very general postulates:

- For arbitrary  $j$  the right  $(j, 0)$  and the left  $(0, j)$  handed spinors transform according to the Wigner's rules [26, 16]:

$$\phi_R(p^\mu) = \Lambda_R(p^\mu \leftarrow \overset{\circ}{p}^\mu) \phi_R(\overset{\circ}{p}^\mu) = \exp(+\mathbf{J} \cdot \boldsymbol{\varphi}) \phi_R(\overset{\circ}{p}^\mu) \quad , \quad (5)$$

$$\phi_L(p^\mu) = \Lambda_L(p^\mu \leftarrow \overset{\circ}{p}^\mu) \phi_L(\overset{\circ}{p}^\mu) = \exp(-\mathbf{J} \cdot \boldsymbol{\varphi}) \phi_L(\overset{\circ}{p}^\mu) \quad . \quad (6)$$

$\Lambda_{R,L}$  are the matrices of Lorentz boosts;  $\mathbf{J}$  are the spin matrices for spin  $j$ ;  $\boldsymbol{\varphi}$  are parameters of the given boost. If restrict ourselves by the case of bradyons they are defined, *e. g.*, refs. [27, 17], by means of:

$$\cosh(\varphi) = \gamma = \frac{1}{\sqrt{1-v^2}} = \frac{E}{m}, \quad \sinh(\varphi) = v\gamma = \frac{|\mathbf{p}|}{m}, \quad \hat{\boldsymbol{\varphi}} = \mathbf{n} = \frac{\mathbf{p}}{|\mathbf{p}|} \quad . \quad (7)$$

- $\phi_L$  and  $\phi_R$  are the eigenspinors of the helicity operator  $(\mathbf{J} \cdot \mathbf{n})$ :

$$(\mathbf{J} \cdot \mathbf{n}) \phi_{R,L}(p^\mu) = h \phi_{R,L}(p^\mu) \quad (8)$$

( $h = -j, -j+1, \dots, j$  is the helicity quantum number).

- The relativistic dispersion relations  $E = \pm \sqrt{\mathbf{p}^2 + m^2}$  are hold for observed particle states.

Apart from these mathematical postulates the cornerstone of relativistic theories appears to be the Ryder-Burgard (RB) relation. Ryder [27] writes: “When a particle is at rest, one cannot define its spin as either left- or right-handed, so  $\phi_R(0) = \phi_L(0)$ .” In order to include negative-energy solutions it is necessary to take into account the possibility of opposite sign in the relation for right- and left- spinors [17]. On the base of the RB relation and the postulates which is above (the Wigner rules for the Lorentz transformations) the Dirac equation follows immediately, ref. [20b]. “Refer to Eqs. (5) and (6) and set  $\mathbf{J} = \boldsymbol{\sigma}/2$ . Next, note that spinors [implied by the arguments based on parity symmetry and that Lorentz group is essentially  $SU_R(2) \otimes SU_L(2)$ ]

$$\psi(p^\mu) = \begin{pmatrix} \phi_R(p^\mu) \\ \phi_L(p^\mu) \end{pmatrix} \quad (9)$$

turn out to be of crucial significance in constructing a field  $\Psi(x)$  that describes eigenstates of the Charge operator,  $Q$ , if

$$\phi_R(\overset{\circ}{p}^\mu) = \pm \phi_L(\overset{\circ}{p}^\mu) \quad (10)$$

(otherwise physical eigenstates are no longer charge eigenstates). We call [this relation], the “Ryder-Burgard relation”... Next couple the RB relation with Eqs. (5) and (6) to obtain

$$\begin{pmatrix} \mp m \mathbf{1} & p_0 + \boldsymbol{\sigma} \cdot \mathbf{p} \\ p_0 - \boldsymbol{\sigma} \cdot \mathbf{p} & \mp m \mathbf{1} \end{pmatrix} \psi(p^\mu) = 0 \quad . \quad (11)$$

[Above we have used the property

$$[\Lambda_{L,R}(p^\mu \leftarrow \overset{\circ}{p}^\mu)]^{-1} = [\Lambda_{R,L}(p^\mu \leftarrow \overset{\circ}{p}^\mu)]^\dagger \quad (12)$$

and that both  $\mathbf{J}$  and  $\Lambda_{R,L}$  are Hermitian for the finite, *e.g.*,  $(1/2, 0) \oplus (0, 1/2)$  representation of the Lorentz group]. Introducing  $\Psi(x) \equiv \psi(p^\mu) \exp(\mp i p \cdot x)$  and letting  $p_\mu \rightarrow i\partial_\mu$ , the above equation becomes:  $(i\gamma^\mu \partial_\mu - m \mathbf{1})\Psi(x) = 0$ . This is the Dirac equation for spin-1/2 particles with  $\gamma^\mu$  in the Weil/Chiral representation. Similarly, one can obtain wave equations and thus a complete kinematic structure and the associated dynamical consequences for other Dirac-like  $(j, 0) \oplus (0, j)$  spinors  $\psi(p^\mu)$  and quantum fields  $\Psi(x)$ .”

Let me now consider generalized cases.

1.

$$\phi_R^\pm(\overset{\circ}{p}^\mu) = \mathcal{A} \phi_L^\pm(\overset{\circ}{p}^\mu) \quad . \quad (13)$$

$\mathcal{A}$  is the matrix of arbitrary linear transformation. Expanding it in the complete set of  $\sigma_i$  matrices ( $\mathcal{A} \equiv \mathbf{1} c_1^0 + \boldsymbol{\sigma} \cdot \mathbf{c}_1$ ) one can easily obtain:

$$\phi_R^\pm(\overset{\circ}{p}^\mu) = e^{i\alpha_\pm} \phi_L^\pm(\overset{\circ}{p}^\mu) \quad . \quad (14)$$

We have used the second postulate. In spite of the fact that, in general,  $\mathbf{c}_1 \not\parallel \mathbf{p}$  this is possible for  $\phi_{R,L}(\overset{\circ}{p}^\mu)$  spinors as was explained in ref. [28, p.93].

The way of deriving the equation is the same to the above:

$$\begin{aligned}\phi_R^\pm(p^\mu) &= \Lambda_R(p^\mu \leftarrow \hat{p}^\mu) \phi_R^\pm(\hat{p}^\mu) = e^{i\alpha_\pm} \Lambda_R(p^\mu \leftarrow \hat{p}^\mu) \phi_L^\pm(\hat{p}^\mu) = \\ &= e^{i\alpha_\pm} \Lambda_R(p^\mu \leftarrow \hat{p}^\mu) \Lambda_L^{-1}(p^\mu \leftarrow \hat{p}^\mu) \phi_L^\pm(p^\mu) \quad ,\end{aligned}\quad (15)$$

$$\begin{aligned}\phi_L^\pm(p^\mu) &= \Lambda_L(p^\mu \leftarrow \hat{p}^\mu) \phi_L^\pm(\hat{p}^\mu) = e^{-i\alpha_\pm} \Lambda_L(p^\mu \leftarrow \hat{p}^\mu) \phi_R^\pm(\hat{p}^\mu) = \\ &= e^{-i\alpha_\pm} \Lambda_L(p^\mu \leftarrow \hat{p}^\mu) \Lambda_R^{-1}(p^\mu \leftarrow \hat{p}^\mu) \phi_R^\pm(p^\mu) \quad .\end{aligned}\quad (16)$$

Using definitions of the Lorentz boost (5-7) one can re-write the equations (15,16) in matrix form (provided that  $m \neq 0$ ):

$$\begin{pmatrix} -me^{-i\alpha_\pm} & p_0 + (\boldsymbol{\sigma} \cdot \mathbf{p}) \\ p_0 - (\boldsymbol{\sigma} \cdot \mathbf{p}) & -me^{i\alpha_\pm} \end{pmatrix} \begin{pmatrix} \phi_R(p^\mu) \\ \phi_L(p^\mu) \end{pmatrix} = 0 \quad ,\quad (17)$$

or

$$(\hat{p} - mT) \psi(p^\mu) = 0 \quad ,\quad (18)$$

with

$$T = \begin{pmatrix} e^{-i\alpha_\pm} & 0 \\ 0 & e^{i\alpha_\pm} \end{pmatrix} \quad .\quad (19)$$

Particular cases are:

$$\alpha_\pm = 0, 2\pi \quad : \quad (\hat{p} - m)\psi(p^\mu) = 0 \quad ,\quad (20)$$

$$\alpha_\pm = \pm \pi \quad : \quad (\hat{p} + m)\psi(p^\mu) = 0 \quad ,\quad (21)$$

$$\alpha_\pm = +\pi/2 \quad : \quad (\hat{p} + im\gamma_5)\psi(p^\mu) = 0 \quad ,\quad (22)$$

$$\alpha_\pm = -\pi/2 \quad : \quad (\hat{p} - im\gamma_5)\psi(p^\mu) = 0 \quad .\quad (23)$$

Equations (20,21) are the well-known Dirac equations for positive- and negative-energy bispinors in the momentum space. Equations of the type (22,23) had also been discussed in the old literature, *e. g.*, ref. [31]. They have been named as the Dirac equations for 4-spinors of the second kind [29, 30, 20]. Their possible relevance to describing neutrino had been mentioned in the cited papers.

2. Another generalization deals with the RB relation in the other form, which is not equivalent to the first one (let me remind that complex conjugation is not a linear operator):

$$\phi_R(\hat{p}^\mu) = \mathcal{B} \phi_L^*(\hat{p}^\mu) \quad .\quad (24)$$

In this case we use expansion in the different complete set of  $\sigma_i$  matrices (let me remind a mathematical theorem that after multiplying each of matrices, which form a complete set, by a non-singular matrix the property to be the complete set is hold). We come to the equation (24) re-written in convenient form:

$$\begin{aligned}\phi_R^\pm(\hat{p}^\mu) &= \mathcal{B}[\phi_L^\pm(\hat{p}^\mu)]^* = [c_2^0 \sigma_2 + (\boldsymbol{\sigma} \cdot \mathbf{c}_2) \sigma_2] [\phi_L^\pm(\hat{p}^\mu)]^* = \\ &= [i c_2^0 \Theta_{[1/2]} \mp i(|\Re \mathbf{c}_2| + i|\Im \mathbf{c}_2|) \Theta_{[1/2]}] [\phi_L^\pm(\hat{p}^\mu)]^* \\ &= i e^{i\beta_\mp} \Theta_{[1/2]} [\phi_L^\pm(\hat{p}^\mu)]^*\end{aligned}\quad (25)$$

and, hence, to the inverse one

$$\phi_L^\pm(\hat{p}^\mu) = -i e^{i\beta_\mp} \Theta_{[1/2]} [\phi_R^\pm(\hat{p}^\mu)]^* \quad . \quad (26)$$

We have used above that  $\sigma_2$  matrix is connected with the Wigner operator  $\Theta_{[1/2]} = -i\sigma_2$  and the property of the Wigner operator for any spin  $\Theta_{[j]} \mathbf{J} \Theta_{[j]}^{-1} = -\mathbf{J}^*$ . So, if  $\phi_{L,R}$  is an eigenstate of the helicity operator, then  $\Theta_{[j]} \phi_{L,R}^*$  is the eigenstate with the opposite helicity quantum number:

$$(\mathbf{J} \cdot \mathbf{n}) \Theta_{[j]} \left[ \phi_{L,R}^h(p^\mu) \right]^* = -h \Theta_{[j]} \left[ \phi_{L,R}^h(p^\mu) \right]^* \quad . \quad (27)$$

Therefore, from Eqs. (25,26) we have

$$\phi_R^\pm(p^\mu) = +i e^{i\beta_\mp} \Lambda_R(p^\mu \leftarrow \hat{p}^\mu) \Theta_{[1/2]} [\Lambda_L^{-1}(p^\mu \leftarrow \hat{p}^\mu)]^* [\phi_L^\pm(p^\mu)]^* \quad , \quad (28)$$

$$\phi_L^\pm(p^\mu) = -i e^{i\beta_\mp} \Lambda_L(p^\mu \leftarrow \hat{p}^\mu) \Theta_{[1/2]} [\Lambda_R^{-1}(p^\mu \leftarrow \hat{p}^\mu)]^* [\phi_R^\pm(p^\mu)]^* \quad . \quad (29)$$

Using the mentioned property of the Wigner operator we transform Eqs. (28,29) to

$$\phi_R^\pm(p^\mu) = +i e^{i\beta_\mp} \Theta_{[1/2]} [\phi_L^\pm(p^\mu)]^* \quad , \quad (30)$$

$$\phi_L^\pm(p^\mu) = -i e^{i\beta_\mp} \Theta_{[1/2]} [\phi_R^\pm(p^\mu)]^* \quad . \quad (31)$$

Finally, in matrix form one has

$$\begin{pmatrix} \phi_R(p^\mu) \\ \phi_L(p^\mu) \end{pmatrix} = e^{i\beta_\mp} \begin{pmatrix} 0 & i\Theta_{[1/2]} \\ -i\Theta_{[1/2]} & 0 \end{pmatrix} \begin{pmatrix} \phi_R^*(p^\mu) \\ \phi_L^*(p^\mu) \end{pmatrix} = S_{[1/2]}^c \begin{pmatrix} \phi_R(p^\mu) \\ \phi_L(p^\mu) \end{pmatrix} \quad , \quad (32)$$

with  $S_{[1/2]}^c$  being the operator of charge conjugation in the  $(1/2, 0) \oplus (0, 1/2)$  representation space, *e.g.*, ref. [32]. We obtain, in fact, conditions of self/anti-self charge conjugacy:

$$\psi(p^\mu) = \pm \psi^c(p^\mu) \quad . \quad (33)$$

Thus, depending on relations between left- and right-handed spinors (as a matter of fact, depending on the choice of the spinorial basis) we can describe physical excitations of the very different physical nature.

3. The most general form of the RB relation is:

$$\phi_R(\hat{p}^\mu) = \mathcal{A} \phi_L(\hat{p}^\mu) + \mathcal{B} \phi_L^*(\hat{p}^\mu) \quad , \quad (34)$$

The generalized form of the equation in the  $(1/2, 0) \oplus (0, 1/2)$  representation space is then:

$$\left[ a \frac{\hat{p}}{m} + b \mathcal{T} S_{[1/2]}^c - \mathcal{T} \right] \psi(p^\mu) = 0 \quad , \quad a^2 + b^2 = 1 \quad . \quad (35)$$

Using computer algebra systems, *e.g.*, MATHEMATICA 2.2 it is easy to check that the equation has correct relativistic dispersion relations (see the third item of the set of postulates).

The following part of my talk is concerned with the ‘old-fashioned’ formalism proposed by Professor S. Weinberg long ago, namely, the  $2(2j+1)$  formalism [33]-[35]. In spite of some antiquity of this formalism, in our opinion, it does not deserve ‘to be retired’. The equation for a  $j=1$  case, proposed in the sixties, is:

$$[\gamma_{\mu\nu} p^\mu p^\nu - m^2 \mathbf{1}] \Psi(x) = 0 \quad , \quad (36)$$

where  $\gamma^{\mu\nu}$  are the Barut-Muzinich-Williams covariantly defined matrices;  $p_\mu = i\partial_\mu$ . However, for the Dirac-like states the equation has been corrected recently [17]:

$$[\gamma_{\mu\nu} \partial^\mu \partial^\nu + \wp_{u,v} m^2 \mathbf{1}] \Psi(x) = 0 \quad , \quad (37)$$

$\wp_{u,v} = \pm 1$ . The cited work [17] presents itself a realization of the quantum field theory of the Bargmann-Wightman-Wigner (BWW) type [16] in the  $(1,0) \oplus (0,1)$  representation space. One can follow the same procedure which has been applied above in the  $(1/2,0) \oplus (0,1/2)$  representation. For a  $j=1$  case the first generalization then yields:

$$[\gamma_{\mu\nu} p^\mu p^\nu - \mathcal{T} m^2] \psi(p^\mu) = 0 \quad . \quad (38)$$

The second one is:

$$\psi(p^\mu) = \Gamma_5 S_{[1]}^c \psi(p^\mu) \quad , \quad (39)$$

with

$$S_{[1]}^c = \begin{pmatrix} 0 & \Theta_{[1]} \\ -\Theta_{[1]} & 0 \end{pmatrix} \mathcal{K} \quad , \quad (40)$$

and  $\mathcal{K}$  is the complex conjugation operator. As opposed to a spin-1/2 case, we have now the condition of  $\Gamma_5 S_{[1]}^c$ -conjugacy. So, depending on the choice of representation space, some subtle mathematical differences may arise.

I want to do several remarks: 1) In a spin-1 case the wave function (field operator)  $\Psi(x)$  can be re-written in the bivector form or the antisymmetric tensor form, ref. [14]; 2) The spin structure of matrix elements for interaction of two  $j=1$  Joos-Weinberg particles with vector potential is very similar to a spin-1/2 case, ref. [36]. This is certainly an advantage, because this fact permits us to use many calculations produced for the well-developed fermion theory; 3) If we use only one field  $\Psi(x)$ , the  $j=1$  Hamiltonian operator is energy-dependent; the Feynman-Dyson propagator is not equal to the Wick propagator; interaction with the external vector potential leads to unrenormalizable theory; the theory contains unexplained tachyonic solutions, *etc.* They are, of course, shortcomings from a viewpoint of modern theory.

Let me consider now models with spinors of the second kind. Namely, the phase factor takes values  $\alpha = \pm \frac{\pi}{2}$ , or the Ryder-Burgard relation takes a form:  $\phi_R(\hat{p}^\mu) = \pm i \phi_L(\hat{p}^\mu)$ . Equations in the momentum representation follow immediately:

$$[i\gamma_5 \hat{p} - m] \Upsilon_\pm(p^\mu) = 0 \quad , \quad (41)$$

$$[i\gamma_5 \hat{p} + m] \mathcal{B}_\pm(p^\mu) = 0 \quad . \quad (42)$$

Then, equations in the coordinate representation are

$$[\gamma^5 \gamma^\mu \partial_\mu + m] \Psi_{(1)}(x) = 0 \quad , \quad \bar{\Psi}_{(1)}(x) [\gamma^5 \gamma^\mu \partial_\mu + m] = 0 \quad . \quad (43)$$

They lead to the following theorem, which is easily proved in a straightforward manner.

**Theorem:** One can *not* construct the Lagrangian in terms of independent field variables  $\Psi_{(1)}$  and  $\bar{\Psi}_{(1)}$ , that could lead to the Lagrange-Euler equations of the form (43).

If we still wish to construct the Lagrangian we are forced to introduce another field satisfying the equations:

$$[\gamma^5 \gamma^\mu \partial_\mu - m] \Psi_{(2)}(x) = 0 \quad , \quad \bar{\Psi}_{(2)}(x) [\gamma^5 \gamma^\mu \partial_\mu - m] = 0 \quad . \quad (44)$$

Their characteristic feature is the opposite sign at the mass term (comparing with  $\Psi_{(1)}(x)$ ) in the coordinate representation (*cf.* also with new models in the  $(1,0) \oplus (0,1)$  space). Spinors satisfying equations (41,42) are not the eigenspinors of the Parity operator. These facts hint that we obtain another example of the quantum field theory discussed in ref. [16, 17]. The corresponding Lagrangian can be written as follows:

$$\begin{aligned} \mathcal{L} = & \frac{1}{2} [\bar{\Psi}_2 \gamma^\mu \gamma^5 \partial_\mu \Psi_1 + \partial_\mu \bar{\Psi}_1 \gamma^\mu \gamma^5 \Psi_2 - \bar{\Psi}_1 \gamma^\mu \gamma^5 \partial_\mu \Psi_2 - \partial_\mu \bar{\Psi}_2 \gamma^\mu \gamma^5 \Psi_1] - \\ & - m [\bar{\Psi}_1 \Psi_2 + \bar{\Psi}_2 \Psi_1] \quad . \end{aligned} \quad (45)$$

Physical consequences of this model are the following:

- Depending on relations between creation and annihilation operators of  $\Psi_{(1)}$  and  $\Psi_{(2)}$  we can describe charged particles in the  $(1/2, 0) \oplus (0, 1/2)$  representation space, but also one can obtain neutral particles.
- Formalism admits (in particular cases) the use of either commutation or anticommutation relations. One can describe bosons in the  $(1/2, 0) \oplus (0, 1/2)$  representation space.
- There is a puzzled physical ‘excitation’ with  $E \equiv 0$ ,  $Q \equiv 0$  and  $(W \cdot n) \equiv 0$ .
- Transitions  $\Psi_{(1)} \leftrightarrow \Psi_{(2)}$  are possible. In order to calculate contributions to self-energies and/or vertex functions development of the Feynman diagram technique (or other methods of higher order calculations) is required.
- The role of the Feynman-Dyson propagators is similar to the Dirac theory: to propagate positive-frequency solutions toward positive times and the negative-frequency ones, backward in time, *i.e.*, it is compatible with the Feynman-Stückelberg scheme. But, the Feynman-Dyson propagators are not the Wick propagators (*cf.* with a  $j = 1$  case).
- As a result of analysis of this model the question arises: if start from the conventional Dirac Lagrangian but vary using another field variables, *e.g.*,  $\gamma_5 \Psi$ ; or  $\frac{1}{\sqrt{2}}(1 - i\gamma_5)\Psi$ , what could we obtain?

One can go further: I present the Majorana-Ahluwalia ideas [19, 20]. We begin with introduction of second-type 4-spinors defined by the formulas:

$$\lambda(p^\mu) \equiv \begin{pmatrix} (\zeta_\lambda \Theta_{[j]}) \phi_L^*(p^\mu) \\ \phi_L(p^\mu) \end{pmatrix} , \quad \rho(p^\mu) \equiv \begin{pmatrix} \phi_R(p^\mu) \\ (\zeta_\rho \Theta_{[j]})^* \phi_R^*(p^\mu) \end{pmatrix} \quad . \quad (46)$$

They are not in helicity eigenstates (see above, Eq. (27)), but one can introduce another quantum number, the chiral helicity,  $\eta$  as in ref. [20]. Phase factors  $\xi_\lambda$  and



$\xi_\rho$  are fixed by the conditions of self/anti-self  $\theta$ -conjugacy:

$$S_{[1/2]}^c \lambda(p^\mu) = \pm \lambda(p^\mu), \quad S_{[1/2]}^c \rho(p^\mu) = \pm \rho(p^\mu) \quad , \quad (47)$$

for a  $j = 1/2$  case; and

$$\left[ \Gamma^5 S_{[1]}^c \right] \lambda(p^\mu) = \pm \lambda(p^\mu) \quad , \quad \left[ \Gamma^5 S_{[1]}^c \right] \rho(p^\mu) = \pm \rho(p^\mu) \quad , \quad (48)$$

for a  $j = 1$  case. Self/anti-self conjugate spinors do not exist for spin-1 in the considered model. Operators of the charge conjugation are defined by formulas (32,40). The wave equations for  $\lambda$  and  $\rho$  spinors, presented by Ahluwalia [20], are inconvenient for physical applications because they have not been written in covariant form:

$$\left( \begin{array}{cc} -\mathbf{1} & \zeta_\lambda \exp(\mathbf{J} \cdot \boldsymbol{\varphi}) \Theta_{[j]} \Xi_{[j]} \exp(\mathbf{J} \cdot \boldsymbol{\varphi}) \\ \zeta_\lambda \exp(-\mathbf{J} \cdot \boldsymbol{\varphi}) \Xi_{[j]}^{-1} \Theta_{[j]} \exp(-\mathbf{J} \cdot \boldsymbol{\varphi}) & -\mathbf{1} \end{array} \right) \lambda(p^\mu) = 0 \quad (49)$$

For the  $\rho$  spinors the equations are obtained by the substitutions  $\xi_\lambda \rightarrow \xi_\rho^*$  and  $\Theta_{[j]} \Xi_{[j]} \leftrightarrow \Xi_{[j]}^{-1} \Theta_{[j]}$ . Zero-momentum 2-spinors used in Eq. (46) are connected as follows

$$[\phi_L^h(\tilde{p}^\mu)]^* = \Xi_{[j]} \phi_L^h(p^\mu) \quad . \quad (50)$$

It is this form of the RB relation which has been used for deriving equations (49). Matrices  $\Xi_{[j]}$  are defined by the formulas

$$\Xi_{[1/2]} = \begin{pmatrix} e^{i\phi} & 0 \\ 0 & e^{-i\phi} \end{pmatrix}, \quad \Xi_{[1]} = \begin{pmatrix} e^{i2\phi} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i2\phi} \end{pmatrix} \quad , \quad (51)$$

with  $\phi$  being the azimuthal angle associated with  $\mathbf{p} \rightarrow \mathbf{0}$ .

Obtained equations appear not to be dynamical equations. One can note

$$\begin{aligned} \Theta_{[1/2]} \Xi_{[1/2]} &= \Xi_{[1/2]}^{-1} \Theta_{[1/2]} = i \frac{\sigma_1 p_2 - \sigma_2 p_1}{\sqrt{(p+p_3)(p-p_3)}} = \\ &= U_+(p^\mu) U_-(p^\mu) = -U_+^{-1}(p^\mu) U_-^{-1}(p^\mu) = -U_\pm^{-1}(p^\mu) U_\pm(\tilde{p}^\mu) = U_\pm^{-1}(\tilde{p}^\mu) U_\pm(p^\mu) \quad , \end{aligned} \quad (52)$$

where  $U_\pm(p^\mu)$  are the  $2 \times 2$  matrices of the unitary transformation to the helicity representation [37] and ref. [38, p.71]:

$$U_+(p^\mu) \sigma_3 U_+^{-1}(p^\mu) = \frac{(\boldsymbol{\sigma} \cdot \mathbf{p})}{p} \quad , \quad U_-^{-1}(p^\mu) \sigma_3 U_-(p^\mu) = -\frac{(\boldsymbol{\sigma} \cdot \mathbf{p})}{p} \quad ; \quad (53)$$

$p = |\mathbf{p}| = \sqrt{E^2 - m^2}$  and  $\tilde{p}^\mu$  is the parity-conjugated momentum. Let us introduce the unitary matrices

$$U_\pm = \begin{pmatrix} U_\pm(\tilde{p}^\mu) & 0 \\ 0 & U_\pm(p^\mu) \end{pmatrix} \quad , \quad \tilde{U}_\pm = \begin{pmatrix} U_\pm(p^\mu) & 0 \\ 0 & U_\pm(\tilde{p}^\mu) \end{pmatrix} \quad , \quad (54)$$

which transform to the new “helicity” representations. In these new representations the equations (49) are presented by:

$$[\zeta\lambda\gamma^5\gamma^0 - \mathbf{1}] \lambda_H(p^\mu) = 0 \quad , \quad (55)$$

$$[\zeta\lambda\gamma^5\gamma^0 + \mathbf{1}] \tilde{\lambda}_H(p^\mu) = 0 \quad . \quad (56)$$

Analogous results can be obtained in the light-front representation. From the analysis of the general parametrization [39, 20] of 2-spinors in terms of the polar  $\theta$  and the azimuthal  $\phi$  angles associated with the vector  $\mathbf{p} \rightarrow 0$  (we use the same symbol  $\xi_{\pm h}$  for  $\phi_L(\hat{p}^\mu)$  and  $\phi_R(\hat{p}^\mu)$  here):

$$\xi_{+1/2} = N e^{i\vartheta_1^{R,L}} \begin{pmatrix} \cos(\theta/2) e^{-i\phi/2} \\ \sin(\theta/2) e^{+i\phi/2} \end{pmatrix} \quad , \quad (57)$$

$$\xi_{-1/2} = N e^{i\vartheta_2^{R,L}} \begin{pmatrix} \sin(\theta/2) e^{-i\phi/2} \\ -\cos(\theta/2) e^{+i\phi/2} \end{pmatrix} \quad , \quad (58)$$

for spin-1/2; and

$$\xi_{+1} = N e^{i\delta_1^{R,L}} \begin{pmatrix} \frac{1}{2}(1 + \cos\theta) e^{-i\phi} \\ \sqrt{\frac{1}{2}} \sin\theta \\ \frac{1}{2}(1 - \cos\theta) e^{+i\phi} \end{pmatrix} \quad , \quad \xi_{-1} = N e^{i\delta_3^{R,L}} \begin{pmatrix} \frac{1}{2}(1 - \cos\theta) e^{-i\phi} \\ -\sqrt{\frac{1}{2}} \sin\theta \\ \frac{1}{2}(1 + \cos\theta) e^{+i\phi} \end{pmatrix} \quad (59)$$

$$\xi_0 = N e^{i\delta_2^{R,L}} \begin{pmatrix} -\sqrt{\frac{1}{2}} \sin\theta e^{-i\phi} \\ \cos\theta \\ \sqrt{\frac{1}{2}} \sin\theta e^{+i\phi} \end{pmatrix} \quad , \quad (60)$$

for spin-1, one can find another forms of the RB relation, connecting 2-spinors of the opposite helicity. They are

$$[\phi_L^h(\hat{p}^\mu)]^* = (-1)^{1/2-h} e^{-i(\theta_1+\theta_2)} \Theta_{[1/2]} \phi_L^{-h}(\hat{p}^\mu) \quad , \quad (61)$$

for a  $j = 1/2$  case; and

$$[\phi_L^h(\hat{p}^\mu)]^* = (-1)^{1-h} e^{-i\delta} \Theta_{[1]} \phi_L^{-h}(\hat{p}^\mu) \quad , \quad (62)$$

for a  $j = 1$  case ( $\delta = \delta_1 + \delta_3$  for  $h = \pm 1$  and  $\delta = 2\delta_2$ , for  $h = 0$ ). As a result of the use of general procedure of deriving wave equations [27, 20] we come to the equations (41,42). Spinors  $\Upsilon_\pm(p^\mu)$  and  $\mathcal{B}_\pm(p^\mu)$  are in helicity eigenstates. They can be presented in the following form

$$\Upsilon_\pm(p^\mu) = \begin{pmatrix} \pm i \Theta_{1/2} [\phi_L^{\mp 1/2}(p^\mu)]^* \\ \phi_L^{\pm 1/2}(p^\mu) \end{pmatrix} \quad , \quad \mathcal{B}_\pm(p^\mu) = \begin{pmatrix} \mp i \Theta_{1/2} [\phi_L^{\mp 1/2}(p^\mu)]^* \\ \phi_L^{\pm 1/2}(p^\mu) \end{pmatrix} \quad , \quad (63)$$

or

$$\tilde{\Upsilon}_\pm(p^\mu) = \begin{pmatrix} \phi_R^{\pm 1/2}(p^\mu) \\ \mp i \Theta_{1/2} [\phi_R^{\mp 1/2}(p^\mu)]^* \end{pmatrix} \quad , \quad \tilde{\mathcal{B}}_\pm(p^\mu) = \begin{pmatrix} \phi_R^{\pm 1/2}(p^\mu) \\ \pm i \Theta_{1/2} [\phi_R^{\mp 1/2}(p^\mu)]^* \end{pmatrix} \quad , \quad (64)$$

if  $\theta_1 + \theta_2 = 0$ . Of course, the latter can differ from the former only by a phase factor provided that we keep the ordinary normalization of 2-spinors. Moreover, one

can note that zero-momentum ‘Dirac-like spinors’ are connected with ‘Majorana-like spinors’:

$$\Upsilon_{+1/2}(\mathring{p}^\mu) = U\lambda_\uparrow^S(\mathring{p}^\mu) = -\gamma_5 U\lambda_\uparrow^A(\mathring{p}^\mu) \quad , \quad (65)$$

$$\mathcal{B}_{+1/2}(\mathring{p}^\mu) = U\lambda_\uparrow^A(\mathring{p}^\mu) = -\gamma_5 U\lambda_\uparrow^S(\mathring{p}^\mu) \quad , \quad (66)$$

$$\Upsilon_{-1/2}(\mathring{p}^\mu) = U\lambda_\downarrow^S(\mathring{p}^\mu) = -\gamma_5 U\lambda_\downarrow^A(\mathring{p}^\mu) \quad , \quad (67)$$

$$\mathcal{B}_{-1/2}(\mathring{p}^\mu) = U\lambda_\downarrow^A(\mathring{p}^\mu) = -\gamma_5 U\lambda_\downarrow^S(\mathring{p}^\mu) \quad . \quad (68)$$

The transformation matrix is

$$U = \begin{pmatrix} \Xi_{[1/2]}^{-1} \Theta_{[1/2]}^{-1} & 0 \\ 0 & \mathbf{1} \end{pmatrix} \quad . \quad (69)$$

Other important relations between arbitrary-momentum ‘Dirac-like’ and ‘Majorana-like’ 4-spinors are:

$$\Upsilon_+(p^\mu) = \pm \frac{1+\gamma_5}{2} \lambda_\downarrow^{S,A}(p^\mu) + \frac{1-\gamma_5}{2} \lambda_\uparrow^{S,A}(p^\mu) \quad , \quad (70)$$

$$\Upsilon_-(p^\mu) = \mp \frac{1+\gamma_5}{2} \lambda_\uparrow^{S,A}(p^\mu) + \frac{1-\gamma_5}{2} \lambda_\downarrow^{S,A}(p^\mu) \quad , \quad (71)$$

$$\mathcal{B}_+(p^\mu) = \mp \frac{1+\gamma_5}{2} \lambda_\downarrow^{S,A}(p^\mu) + \frac{1-\gamma_5}{2} \lambda_\uparrow^{S,A}(p^\mu) \quad , \quad (72)$$

$$\mathcal{B}_-(p^\mu) = \pm \frac{1+\gamma_5}{2} \lambda_\uparrow^{S,A}(p^\mu) + \frac{1-\gamma_5}{2} \lambda_\downarrow^{S,A}(p^\mu) \quad . \quad (73)$$

Analogous relations exist between  $\tilde{\Upsilon}(p^\mu)$   $\tilde{\mathcal{B}}(p^\mu)$  and  $\rho^{S,A}(p^\mu)$  spinors. Finally, treating  $\lambda^S$  and  $\rho^A$  as positive-energy solutions,  $\lambda^A$  and  $\rho^S$  as negative-energy solutions, the wave equations in the coordinate space are written:

$$i\gamma^\mu \partial_\mu \lambda^S(x) - m\rho^A(x) = 0 \quad , \quad (74)$$

$$i\gamma^\mu \partial_\mu \rho^A(x) - m\lambda^S(x) = 0 \quad ; \quad (75)$$

and

$$i\gamma^\mu \partial_\mu \lambda^A(x) + m\rho^S(x) = 0 \quad , \quad (76)$$

$$i\gamma^\mu \partial_\mu \rho^S(x) + m\lambda^A(x) = 0 \quad . \quad (77)$$

As opposed to Eq. (55,56) they are dynamical equations. Dynamical parts of the equations for  $\lambda^{S,A}$  (and  $\rho^{S,A}$ ) spinors are connected with mass parts of  $\rho^{A,S}$  (and  $\lambda^{A,S}$ ) spinors. Equations (74-77) can be written in the 8-component form as follows:

$$[i\Gamma^\mu \partial_\mu - m] \Psi_{(+)}(x) = 0 \quad , \quad (78)$$

$$[i\Gamma^\mu \partial_\mu + m] \Psi_{(-)}(x) = 0 \quad , \quad (79)$$

where

$$\Psi_{(+)}(x) = \begin{pmatrix} \rho^A(x) \\ \lambda^S(x) \end{pmatrix} \quad , \quad \Psi_{(-)}(x) = \begin{pmatrix} \rho^S(x) \\ \lambda^A(x) \end{pmatrix} \quad (80)$$

are *dibispinors*. For purposes of future researches we define the set of  $8 \times 8$ - component  $\Gamma$ - and  $T$ -matrices

$$\Gamma^\mu = \begin{pmatrix} 0 & \gamma^\mu \\ \gamma^\mu & 0 \end{pmatrix} \quad , \quad \Gamma^5 = \begin{pmatrix} \gamma^5 & 0 \\ 0 & \gamma^5 \end{pmatrix} \quad , \quad \mathbf{L}^5 = \begin{pmatrix} \gamma^5 & 0 \\ 0 & -\gamma^5 \end{pmatrix} \quad , \quad (81)$$

$$T_{11} = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{pmatrix} \quad , \quad T_{01} = \begin{pmatrix} 0 & \mathbf{1} \\ \mathbf{1} & 0 \end{pmatrix} \quad , \quad T_{10} = \begin{pmatrix} 0 & \mathbf{1} \\ -\mathbf{1} & 0 \end{pmatrix} \quad . \quad (82)$$

The latter are determined within phase factors (*e.g.*,  $(-1)^k$ , ref. [30]). The useful commutation relation is

$$\mathbf{L}^5 \Gamma^\nu - \Gamma^\nu \mathbf{L}^5 = 0 \quad . \quad (83)$$

The Lagrangian is then given by the formula:

$$\begin{aligned} \mathcal{L}^{(1)} = & \frac{i}{2} \left[ \bar{\Psi}_{(+)} \Gamma^\mu \partial_\mu \Psi_{(+)} - \partial_\mu \bar{\Psi}_{(+)} \Gamma^\mu \Psi_{(+)} + \bar{\Psi}_{(-)} \Gamma^\mu \partial_\mu \Psi_{(-)} - \partial_\mu \bar{\Psi}_{(-)} \Gamma^\mu \Psi_{(-)} \right] \\ & - m \left[ \bar{\Psi}_{(+)} \Psi_{(+)} - \bar{\Psi}_{(-)} \Psi_{(-)} \right] \quad . \end{aligned} \quad (84)$$

It is useful to note that the Lagrangian admits following gradient transformations of the first kind for  $\lambda^{S,A}(x)$  and  $\rho^{S,A}(x)$  spinors:

$$\lambda'(x) \rightarrow (\cos \alpha - i\gamma^5 \sin \alpha) \lambda(x) \quad , \quad (85)$$

$$\bar{\lambda}'(x) \rightarrow \bar{\lambda}(x) (\cos \alpha - i\gamma^5 \sin \alpha) \quad , \quad (86)$$

$$\rho'(x) \rightarrow (\cos \alpha + i\gamma^5 \sin \alpha) \rho(x) \quad , \quad (87)$$

$$\bar{\rho}'(x) \rightarrow \bar{\rho}(x) (\cos \alpha + i\gamma^5 \sin \alpha) \quad . \quad (88)$$

We still note, in general, different gradient transformations of 4-spinors  $\lambda$  and  $\rho$  are possible. In terms of field functions  $\Psi_{(\pm)}(x)$  the equations (85-88) are written

$$\Psi'_{(\pm)}(x) \rightarrow (\cos \alpha + i\mathbf{L}^5 \sin \alpha) \Psi_{(\pm)}(x) \quad , \quad (89)$$

$$\bar{\Psi}'_{(\pm)}(x) \rightarrow \bar{\Psi}_{(\pm)}(x) (\cos \alpha - i\mathbf{L}^5 \sin \alpha) \quad . \quad (90)$$

Local gradient transformations are introduced after “covariantization” of derivatives:

$$\partial_\mu \rightarrow \nabla_\mu = \partial_\mu - ig\mathbf{L}^5 A_\mu \quad , \quad (91)$$

$$A'_\mu(x) \rightarrow A_\mu + \frac{1}{g} \partial_\mu \alpha \quad . \quad (92)$$

This tells us that the states described by the spinors of the second type can possess the axial charge.

The second remark: neither  $\lambda^{S,A}$  nor  $\rho^{S,A}$  are the eigenfunctions of the Hamiltonian in the  $(1/2, 0) \oplus (0, 1/2)$  representation space:

$$\partial_\mu \gamma^\mu \lambda_\eta(x) + \wp_{\uparrow\downarrow} m \lambda_{-\eta}(x) = 0 \quad , \quad (93)$$

$$\partial_\mu \gamma^\mu \rho_\eta(x) + \wp_{\uparrow\downarrow} m \rho_{-\eta}(x) = 0 \quad , \quad (94)$$

(the indices  $\eta$  should be referred to the chiral helicity introduced in the Ahluwalia's paper [20]). Therefore, the matrix element  $\langle \lambda^A(0), \downarrow | \lambda^S(t), \uparrow \rangle$  (and others) have the non-zero values at the time  $t$ , what produces speculations on the possibility of oscillations between self- and anti-self charge conjugate states. But, the wavelength of these oscillations is very small, of the order of the de Broglie wavelength. The only effect, which could be seen, is the average depletion of the flux composed from the pure, *e.g.*,  $\lambda^S$  states.

We should point out relations with other papers, *e.g.*, ref. [22, 23]. Authors presented the model based on the following principles: a  $j = 1/2$  particle is described by  $\Psi_f(x)$ ; its antiparticle, by  $\Psi_{\bar{f}}(x)$ . These wave functions satisfy different equations:

$$[i\gamma^\mu \partial_\mu - m] \Psi_f(x) = 0 \quad , \quad (95)$$

$$[i\gamma^\mu \partial_\mu + m] \Psi_{\bar{f}}(x) = 0 \quad . \quad (96)$$

In the framework of their model the role of the charge conjugation matrix is played by  $\gamma^5$  matrix. As a matter of fact this model is recreation of the ideas of Belinfante, Pauli [40] and, particularly, of Professor M. Markov [41]. Barut-Ziino asymptotically chiral massive fields:

$$\psi_f^{ch} = \frac{\psi_f - \psi_{\bar{f}}}{\sqrt{2}} \quad , \quad \psi_{\bar{f}}^{ch} = \frac{\psi_f + \psi_{\bar{f}}}{\sqrt{2}} \quad (97)$$

have close relations with  $\lambda^A, \rho^A \sim \psi_f^{ch}$ ,  $\lambda^S, \rho^S \sim \psi_{\bar{f}}^{ch}$ . In this connection Barut and Ziino noted on needed modifications of our understanding of the concept of the quantization space [23]: “Such a Fock space should have the manifestly covariant structure

$$\mathcal{F} \equiv \mathcal{F}^0 \otimes S_{in} \quad , \quad (98)$$

where  $\mathcal{F}^0$  is an ordinary Fock space for one *indistinct* type of positive- and negative-energy identical spin- $\frac{1}{2}$  particles (without regard to the proper-mass sign) and  $S_{in}$  is a two-dimensional internal space spanned by the proper-mass eigenstates  $|+m\rangle$ ,  $|-m\rangle$  thus *doubling*  $\mathcal{F}^0$ . This allows [the Dirac-like fields]  $\psi_f$  and  $\psi_{\bar{f}}$  to be *mixed* if a rotation is performed in  $S_{in}$ .” Moreover, the authors of [22, 23] noted, indeed, that the effect of ‘parity violation’ can be explained in the framework of parity-invariant theory. While the states are not, in general, eigenstates of Parity operator the viewpoint of Barut and Ziino is preferable because it lifts the crucial contradiction with relativity: there are no any reasons Nature to consider left- and right-handed frames in unsymmetrical fashion. As a matter of fact, possibility of such constructs (the former case is called *doubling*) has been discovered by Wigner [16], who enumerated the irreducible projective representations of the full Poincarè group (including reflections). The constructs given by Professor D. V. Ahluwalia *et al.* present themselves explicit examples of the theories of such a type. Neutrino and its antineutrino (of the same chirality) could coincide as a particular case of this model.

I would like to finish my talk by the question which I ask you in the beginning, the question of the claimed longitudinality of the antisymmetric tensor field. Authors of previous works treated both “gauge”-invariant Lagrangians and conformal-invariant Lagrangians. Nevertheless, I can not accept so obvious violation of the Correspondence Principle. The contradiction appears to me to be related with the problem

of acausal non-plane wave solutions [12] in the first-order equations of the form (4), ref. [42], and (4.21,4.22) in [34b,p.B888]. In the classical electromagnetic field theory we know that physical field variables are the strengths  $\mathbf{E}$  and  $\mathbf{B}$ . Potentials are used only as a convenient way to calculate the former. Aharonov and Bohm [43] told us that this is not the case in the quantum theory. Potentials appear to have physical significance. However, attempts to construct the quantized electromagnetic theory based on the use of physical variables (in fact, on the  $(1,0) \oplus (0,1)$  representation of the Poincaré group, *i.e.*, on the first principles) have also certain reasons. Moreover, in ref. [44] Professor S. Weinberg proved that in the quantized theory the 4-vector potential  $A_\mu$  is not a 4-vector (!) at all.

One must begin with the general form of the field operator:

$$F^{\mu\nu}(x) = \sum_h \int \frac{d^3\mathbf{p}}{2E_p(2\pi)^3} \left[ F_{h(+)}^{\mu\nu}(\mathbf{p}) a_h(\mathbf{p}) e^{-ip \cdot x} + F_{h(-)}^{\mu\nu}(\mathbf{p}) b_h^\dagger(\mathbf{p}) e^{+ip \cdot x} \right]. \quad (99)$$

Of course, it is very important question, what we understand under  $F_{h(+)}^{\mu\nu}(\mathbf{p})$  and  $F_{h(-)}^{\mu\nu}(\mathbf{p})$ . One can not forget about the possibility of the use of components of the dual tensor.

The Lagrangian is

$$\mathcal{L} = \frac{1}{4}(\partial_\mu F_{\nu\alpha})(\partial^\mu F^{\nu\alpha}) - \frac{1}{2}(\partial_\mu F^{\mu\alpha})(\partial^\nu F_{\nu\alpha}) - \frac{1}{2}(\partial_\mu F_{\nu\alpha})(\partial^\nu F^{\mu\alpha}) + \frac{1}{4}F_{\mu\nu}F^{\mu\nu}. \quad (100)$$

The massless limit ( $m \rightarrow 0$ ) of this Lagrangian is compatible with conformal invariance (and with the “gauge” invariance within the generalized Lorentz condition, see [45, 14]). The Lagrange-Euler equation is then written

$$\frac{1}{2}(\square + m^2)F_{\mu\nu} + (\partial_\mu F_{\alpha\nu})^{,\alpha} - \partial_\nu F_{\alpha\mu}^{,\alpha} = 0, \quad (101)$$

where  $\square = -\partial_\alpha \partial^\alpha$ . Similar equations follow from the Proca theory (one should take into account the Klein-Gordon equation in the final expression):

$$\partial_\alpha F^{\alpha\mu} + m^2 A^\mu = 0, \quad (102)$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu; \quad (103)$$

and from the Weinberg’s  $2(2j+1)$ - component theory, provided that  $\mathbf{E}$  and  $\mathbf{B}$  are treated to be physical variables. They form the  $j=1$  Weinberg’s wave function.

The variation procedure, ref. [46, 47], for rotation:

$$x^{\mu'} = x^\mu + \omega^{\mu\nu} x_\nu \quad (104)$$

leads to

$$\delta F^{\alpha\beta} = \frac{1}{2} \omega^{\kappa\tau} \mathcal{T}_{\kappa\tau}^{\alpha\beta,\mu\nu} F_{\mu\nu}. \quad (105)$$

Generators of infinitesimal transformations are defined as

$$\begin{aligned} \mathcal{T}_{\kappa\tau}^{\alpha\beta,\mu\nu} &= \frac{1}{2} g^{\alpha\mu} (\delta_\kappa^\beta \delta_\tau^\nu - \delta_\tau^\beta \delta_\kappa^\nu) + \frac{1}{2} g^{\beta\mu} (\delta_\kappa^\nu \delta_\tau^\alpha - \delta_\tau^\nu \delta_\kappa^\alpha) + \\ &+ \frac{1}{2} g^{\alpha\nu} (\delta_\kappa^\mu \delta_\tau^\beta - \delta_\tau^\mu \delta_\kappa^\beta) + \frac{1}{2} g^{\beta\nu} (\delta_\kappa^\alpha \delta_\tau^\mu - \delta_\tau^\alpha \delta_\kappa^\mu). \end{aligned} \quad (106)$$

The classical formula for the Pauli-Lyuban'sky operator is obtained immediately ( $\mathbf{n}||\mathbf{p}$ ):

$$(W \cdot n) = m\epsilon^{ijk}n^i \int d^3\mathbf{x} \left[ F_0^j(\partial^\mu F_\mu^k) + F^{\mu k}(\partial_0 F_\mu^j + \partial_\mu F_0^j + \partial^j F_{0\mu}) \right] \quad . \quad (107)$$

Let us remind that massless limit of this theory, ref. [34], is well-defined. One can substitute the field operator in this expression and we can learn, under what constraints the Pauli-Lyuban'sky operator is equal to zero, why in the previous works the conclusion has been done that massless antisymmetric tensor field appears to be longitudinal (?), and what corresponds the longitudinal solution to.

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